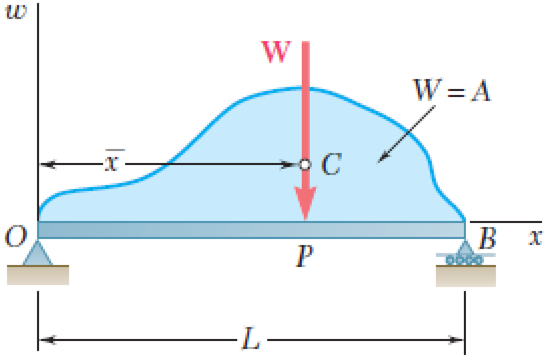
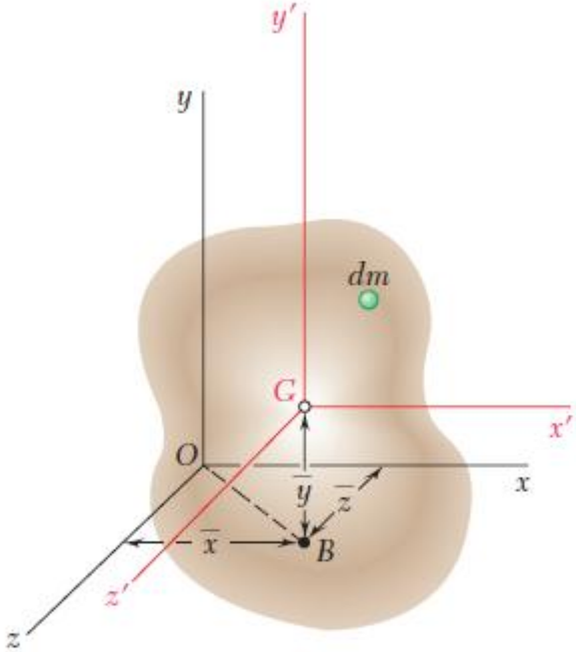
CENTROS DE GRAVEDAD
Y
MPMENTOS DE INERCIA





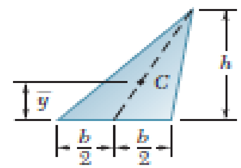
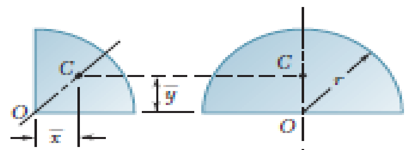
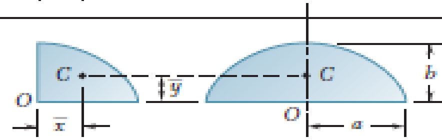
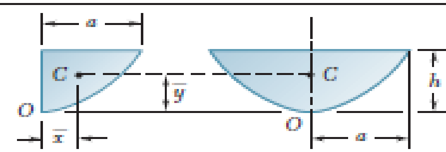
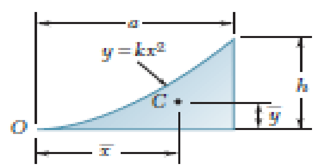
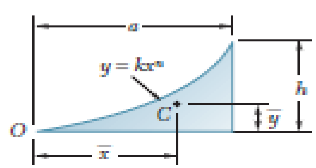
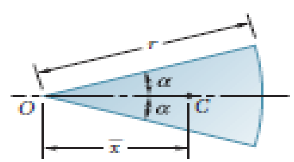
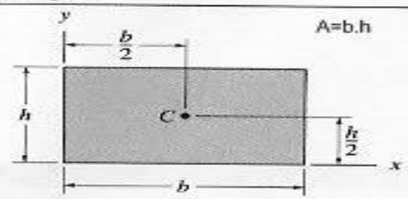
Forma		$\bar{x}$	$\bar{y}$	Área
Área triangular			$\frac{h}{3}$	$\frac{bh}{2}$
Un cuarto de área circular		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Área semicircular		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Un cuarto de área elíptica		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Área semielíptica		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Área semiparabólica		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Área parabólica		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Enjuta parabólica		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Enjuta general		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Sector circular		$\frac{2r \operatorname{sen} \alpha}{3\alpha}$	0	αr^2

Figura 5.8A Centroides de áreas comunes.

Área momento de inercia

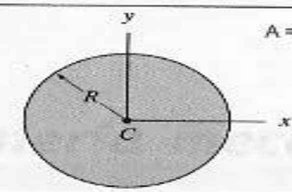
Rectángulo



$\bar{I}_x = \frac{bh^3}{12}$ $\quad \bar{I}_y = \frac{b^3h}{12}$ $\quad \bar{I}_{xy} = 0$

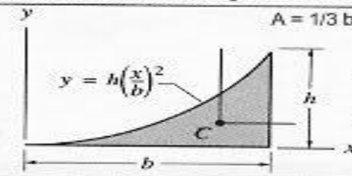
$I_x = \frac{bh^3}{3}$ $\quad I_y = \frac{b^3h}{3}$ $\quad I_{xy} = \frac{b^2h^2}{4}$

Círculo



$I_x = I_y = \frac{\pi R^4}{4}$ $\quad I_{xy} = 0$

Media parabólica complementaria



$y = h\left(\frac{x}{b}\right)^2$

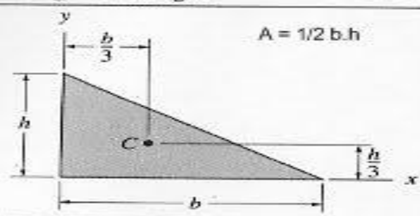
$\bar{x} = \frac{3b}{4}$ $\quad \bar{y} = \frac{3h}{10}$

$\bar{I}_x = \frac{37bh^3}{2100}$ $\quad I_x = \frac{bh^3}{21}$

$\bar{I}_y = \frac{b^3h}{80}$ $\quad I_y = \frac{b^3h}{5}$

$\bar{I}_{xy} = \frac{b^2h^2}{120}$ $\quad I_{xy} = \frac{b^2h^2}{12}$

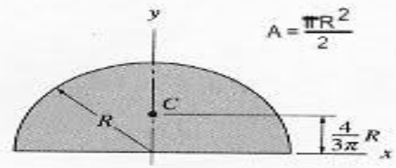
Triángulo rectángulo



$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{36}$ $\quad \bar{I}_{xy} = -\frac{b^2h^2}{72}$

$I_x = \frac{bh^3}{12}$ $\quad I_y = \frac{b^3h}{12}$ $\quad I_{xy} = \frac{b^2h^2}{24}$

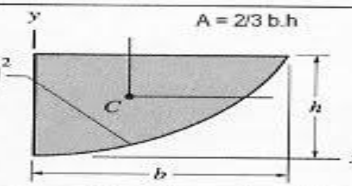
Semicírculo



$\bar{I}_x = 0.1098R^4$ $\quad \bar{I}_{xy} = 0$

$I_x = I_y = \frac{\pi R^4}{8}$ $\quad I_{xy} = 0$

Media parábola



$y = h\left(\frac{x}{b}\right)^2$

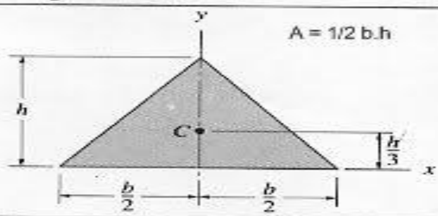
$\bar{x} = \frac{3b}{8}$ $\quad \bar{y} = \frac{3h}{5}$

$\bar{I}_x = \frac{8bh^3}{175}$ $\quad I_x = \frac{2bh^3}{7}$

$\bar{I}_y = \frac{19b^3h}{480}$ $\quad I_y = \frac{2b^3h}{15}$

$\bar{I}_{xy} = \frac{b^2h^2}{60}$ $\quad I_{xy} = \frac{b^2h^2}{6}$

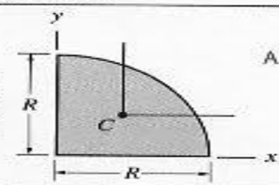
Triángulo isósceles



$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{48}$ $\quad \bar{I}_{xy} = 0$

$I_x = \frac{bh^3}{12}$ $\quad I_{xy} = 0$

Cuarto de círculo

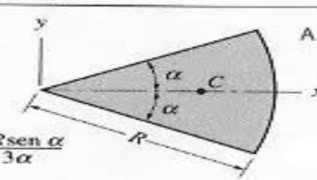


$\bar{x} = \frac{4R}{3\pi}$ $\quad \bar{y} = \frac{4R}{3\pi}$

$\bar{I}_x = \bar{I}_y = 0.054\,88R^4$ $\quad I_x = I_y = \frac{\pi R^4}{16}$

$\bar{I}_{xy} = -0.016\,47R^4$ $\quad I_{xy} = \frac{R^4}{8}$

Sector circular



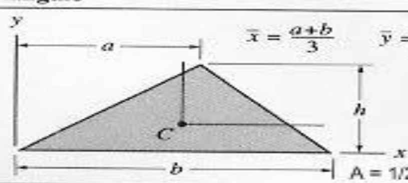
$\bar{x} = \frac{2R \operatorname{sen} \alpha}{3\alpha}$

$I_x = \frac{R^4}{8}(2\alpha - \operatorname{sen} 2\alpha)$

$I_y = \frac{R^4}{8}(2\alpha + \operatorname{sen} 2\alpha)$

$I_{xy} = 0$

Triángulo



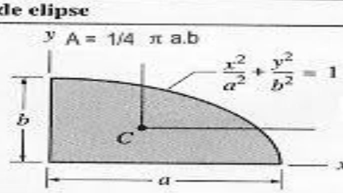
$\bar{x} = \frac{a+b}{3}$ $\quad \bar{y} = \frac{h}{3}$

$\bar{I}_x = \frac{bh^3}{36}$ $\quad I_x = \frac{bh^3}{12}$

$\bar{I}_y = \frac{bh}{36}(a^2 - ab + b^2)$ $\quad I_y = \frac{bh}{12}(a^2 + ab + b^2)$

$\bar{I}_{xy} = \frac{bh^2}{72}(2a - b)$ $\quad I_{xy} = \frac{bh^2}{24}(2a + b)$

Cuarto de elipse



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

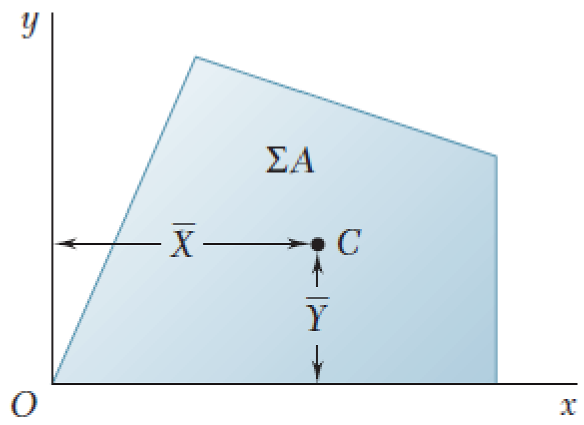
$\bar{x} = \frac{4a}{3\pi}$ $\quad \bar{y} = \frac{4b}{3\pi}$

$\bar{I}_x = 0.054\,88ab^3$ $\quad I_x = \frac{\pi ab^3}{16}$

$\bar{I}_y = 0.054\,88a^3b$ $\quad I_y = \frac{\pi a^3b}{16}$

$\bar{I}_{xy} = -0.016\,47a^2b^2$ $\quad I_{xy} = \frac{a^2b^2}{8}$

CENTROS DE GRAVEDAD DE FIGURAS COMPUESTAS



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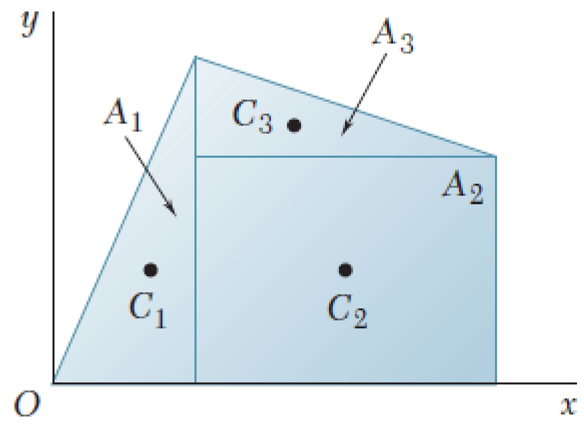


Componente	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³

$$Q_y = \bar{x}A \qquad Q_x = \bar{y}A$$

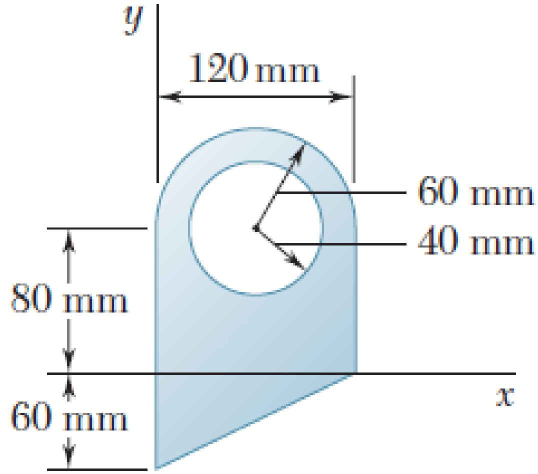
$$I_{x'} = \bar{I}_x + Ad^2$$

Primeros momentos de inercia

Teorema de ejes paralelos de stener

PROBLEMA RESUELTO 5.1

Para el área plana mostrada en la figura, determine: *a*) los primeros momentos con respecto a los ejes *x* y *y*, y *b*) la ubicación de su centroide.



$\bar{X} = 54.8$ mm ◀

$\bar{Y} = 36.6$ mm ◀

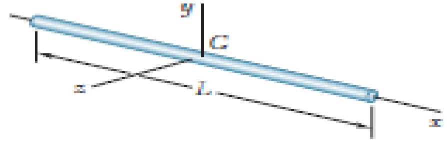
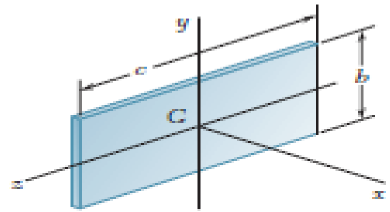
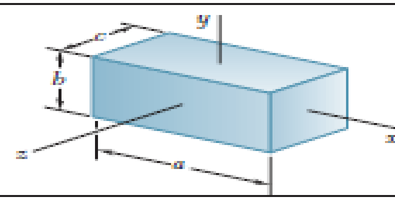
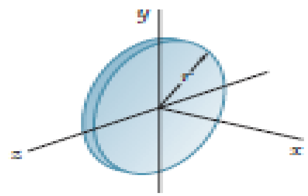
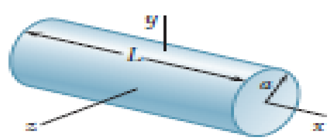
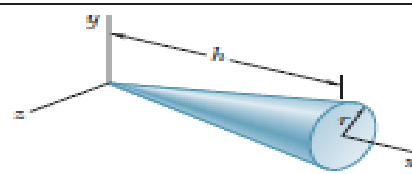
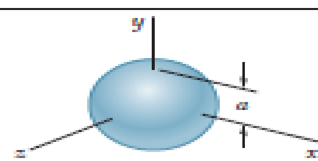
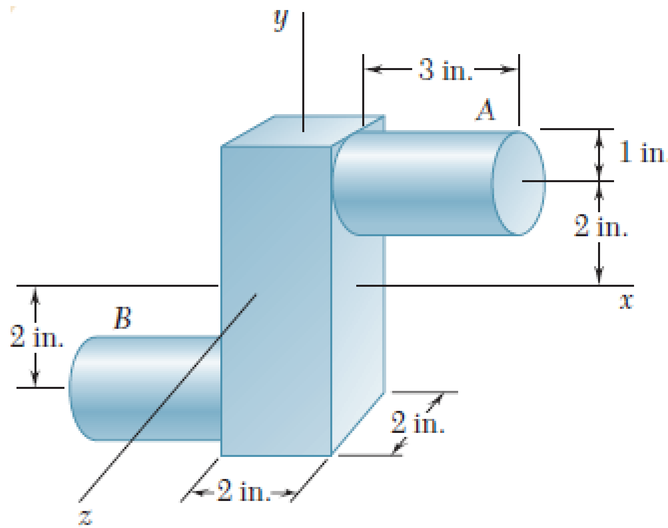
Barra delgada		$I_y = I_z = \frac{1}{12} mL^2$
Placa rectangular delgada		$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} mc^2$ $I_z = \frac{1}{12} mb^2$
Prisma rectangular		$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} m(c^2 + a^2)$ $I_z = \frac{1}{12} m(a^2 + b^2)$
Disco delgado		$I_x = \frac{1}{2} mr^2$ $I_y = I_z = \frac{1}{4} mr^2$
Cilindro circular		$I_x = \frac{1}{2} ma^2$ $I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$
Cono circular		$I_x = \frac{3}{10} ma^2$ $I_y = I_z = \frac{3}{5} m(\frac{1}{4} a^2 + h^2)$
Esfera		$I_x = I_y = I_z = \frac{2}{5} ma^2$

Figura 9.28 Momentos de inercia de masa de formas geométricas comunes.

PROBLEMA RESUELTO 9.12

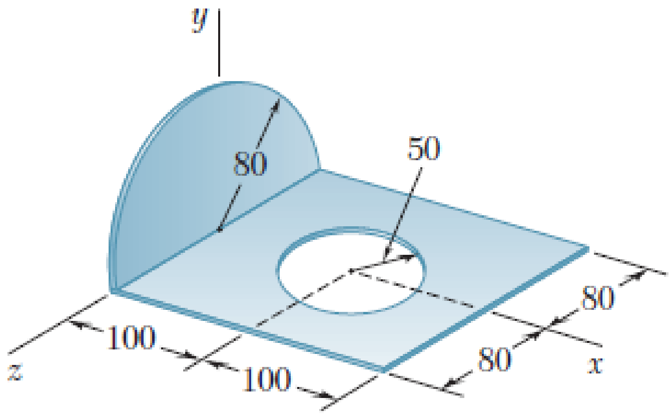
Una pieza de acero consta de un prisma rectangular de 6 × 2 × 2 in. y dos cilindros de 2 in. de diámetro y 3 in. de longitud, como se muestra en la figura. Si se sabe que el peso específico del acero es de 490 lb/ft^3, determine los momentos de inercia de la pieza con respecto a los ejes coordenados.



$I_x = 10.06 \times 10^{-3}$ lb · ft · s^2 ◀
$I_y = 9.32 \times 10^{-3}$ lb · ft · s^2 ◀
$I_z = 17.84 \times 10^{-3}$ lb · ft · s^2 ◀

PROBLEMA RESUELTO 9.13

Una placa delgada de acero de 4 mm de espesor se corta y se dobla para formar la pieza de maquinaria mostrada en la figura. Si se sabe que la densidad del acero es 7 850 kg/m^3, determine los momentos de inercia de la pieza con respecto a los ejes coordenados.



Dimensiones en mm

$$I_x = 3.00 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad ◀$$
$$I_y = 13.28 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad ◀$$
$$I_z = 11.29 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad ◀$$